COM-711 Homework 4

Kevin Smith

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This assignment is due **December 2, 2011**. Turn in your work electronically to Kevin Smith at `kevin.smith@lmc.biol.ethz.ch`. You may submit your work as a PDF file or as a scanned handwritten document.

1 Derive the recursive Bayesian filtering distribution

In lecture 5, we discussed how to arrive at the formula underlying probabilistic tracking algorithms including the particle filter and the Kalman filter. Your task is to start from the joint relation

$$p(X_t, Z_t) = p(\mathbf{x}_t, \mathbf{z}_t, X_{t-1}, Z_{t-1}), \tag{1}$$

and derive the recursive Bayesian filtering distribution, as we did in class. It is given by

$$p(\mathbf{x}_t|Z_t) \propto p(\mathbf{z}_t|\mathbf{x}_t) \int_{\mathbf{x}_{t-1}} p(\mathbf{x}_t|\mathbf{x}_{t-1}) p(\mathbf{x}_{t-1}|Z_{t-1}), \tag{2}$$

where $\mathbf{x}_t$ is the state of the object(s) at time step t, $\mathbf{z}_t$ is the observation at time t, $X_t = \{\mathbf{x}_1, \ldots, \mathbf{x}_{t-1}, \mathbf{x}_t\}$ is the set of all object(s) states, and $Z_t = \{\mathbf{z}_1, \ldots, \mathbf{z}_{t-1}, \mathbf{z}_t\}$ is the set of all current and previous observations. Keep in mind our assumptions: that the dynamic process is Markovian (the current state depends only on the previous state)

$$p(\mathbf{x}_t|X_{t-1}) = p(\mathbf{x}_t|\mathbf{x}_{t-1}), \tag{3}$$

and that the current observation is independent from the other observations, as well as the previous states of the object

$$p(\mathbf{z}_t|\mathbf{x}_t, X_{t-1}, Z_{t-1}) = p(\mathbf{z}_t|\mathbf{x}_t). \tag{4}$$

You should be familiar with the conditional probability relation, and the Champman-Kolmogorov equation (marginalization). Be sure to describe in words what you do in each step of the derivation.

Important note: it is also possible to derive the recursive Bayesian filtering distribution starting from applying Bayes rule to the target posterior:

$$p(\mathbf{x}_t|Z_t) = \frac{p(\mathbf{z}_t|\mathbf{x}_t, Z_{t-1})\; p(\mathbf{x}_t|Z_{t-1})}{p(\mathbf{z}_t|Z_{t-1})}.$$

However, this is not the assignment and **using this approach will result in a loss of points** because it is trivial to find and copy this derivation of the recursive Bayesian filtering equation from the Internet.